Teorema bisectoarei

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Liceul Teoretic W. Shakespeare
Timisoara

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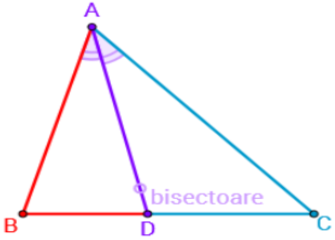
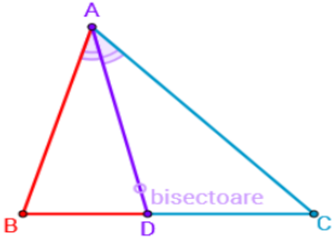


Theorem

Bisectoarea unui unghi al unui triunghi determină pe latura opusă segmente proporționale cu lungimile laturilor ce formează unghiul.



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$\frac{BD}{DC} = \frac{AB}{AC}$

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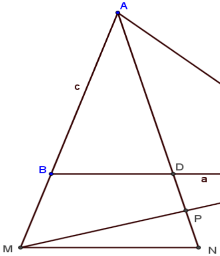

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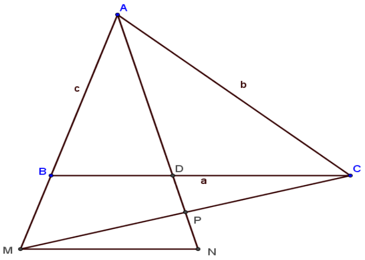

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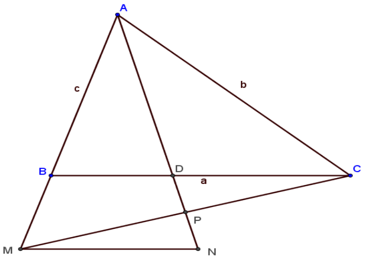

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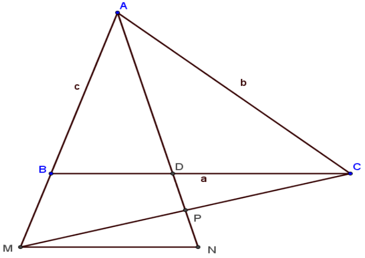

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$\triangle ABD \sim \triangle AMN$ □

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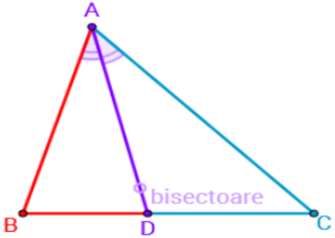

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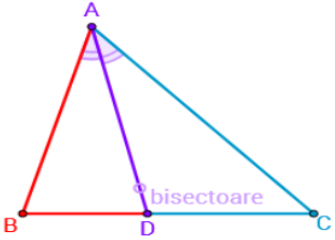
$(AD$ − *bisectoarea* $\angle A \Leftrightarrow$
$\overline{AD} = \frac{b\overline{AB}+c\overline{AC}}{b+c}$



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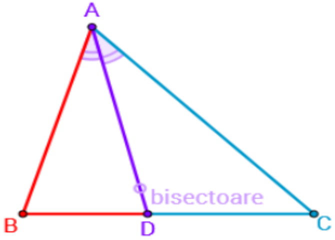

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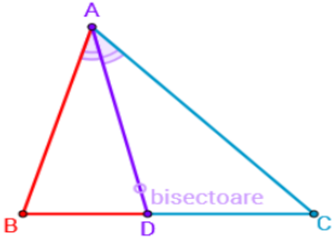

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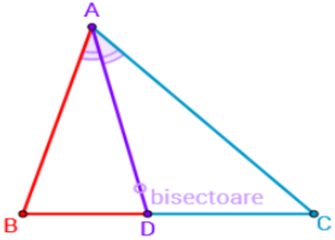

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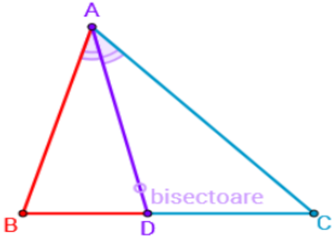

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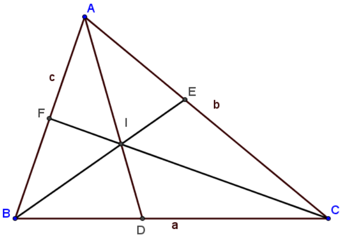

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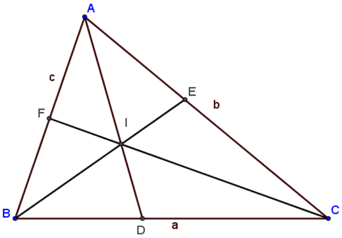

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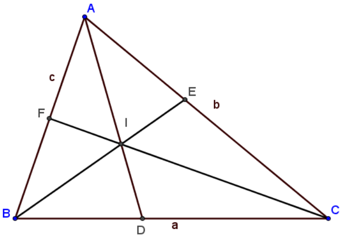

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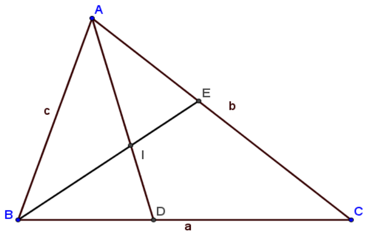
$(CF$ – bisectoarea $\angle C \Leftrightarrow$

$\overline{CF} = \frac{a\overline{CA}+B\overline{CB}}{a+b}$

Teorema 1



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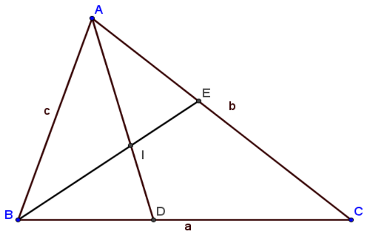

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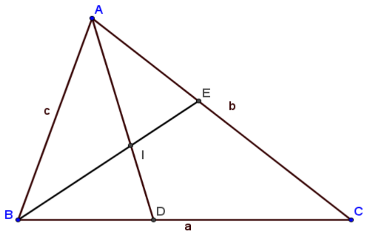

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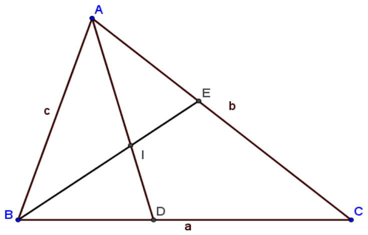

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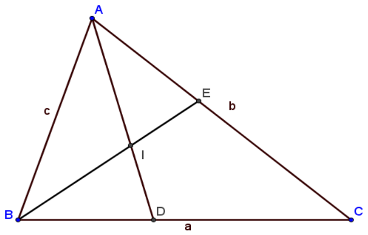

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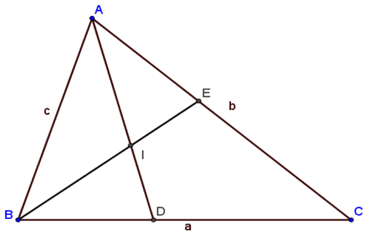

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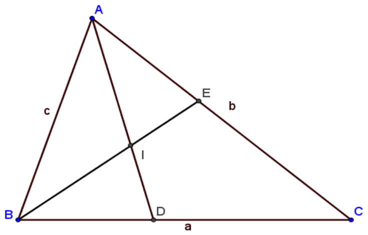

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$\alpha = \frac{b+c}{a+b+c} \Rightarrow \overline{AI} = \frac{b\overline{AB}+c\overline{AC}}{a+b+c}$. □

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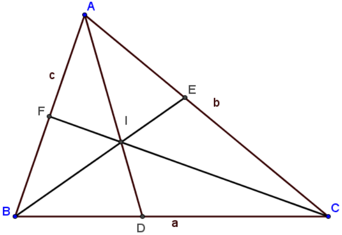

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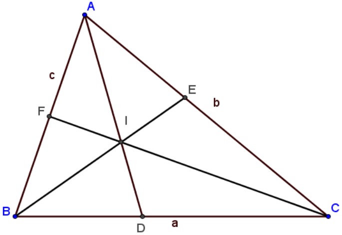

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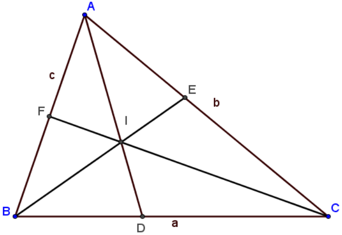

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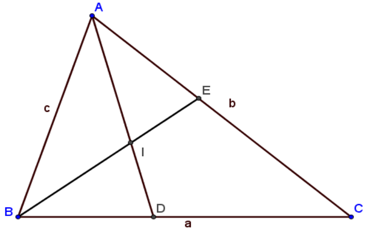
Vectorul de poziție al centrului cercului înscris



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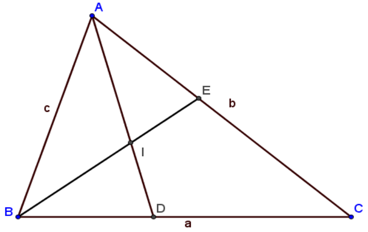

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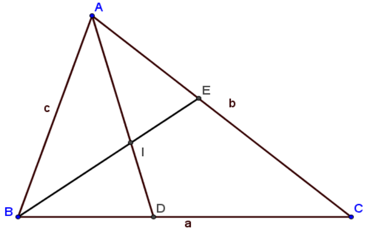

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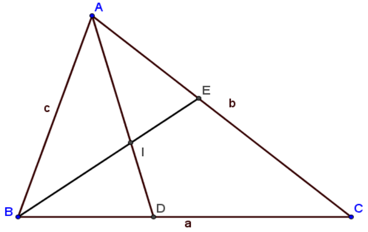

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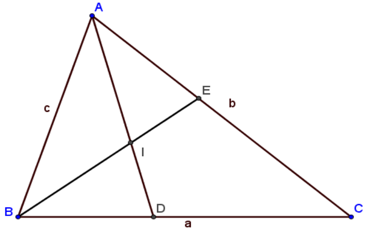

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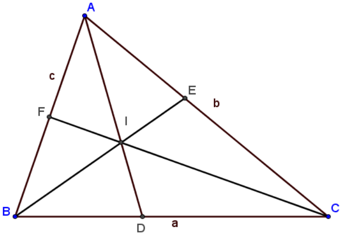
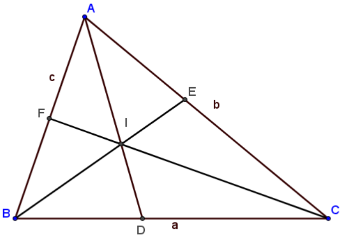
□

Consecințe



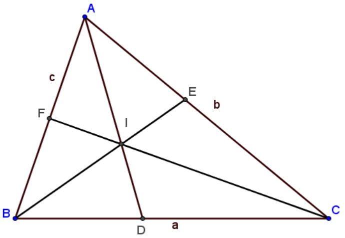
- Într-un triunghi bisectoarele sunt concurente.

Consecințe



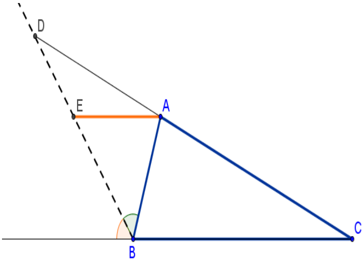
- Într-un triunghi bisectoarele sunt concurente.
- $a\overline{AI} + b\overline{BI} + c\overline{CI} = \overline{0}$

Aplicația 1



În triunghiul $\triangle ABC$, $(AD),(BE),(CF)$ sunt bisectoarele unghiurilor. Arătați că dacă $\overline{AD}+\overline{BE}+\overline{CF}=\overline{0}$, atunci $\triangle ABC$ este echilateral.

Teorema bisectoarei exterioare



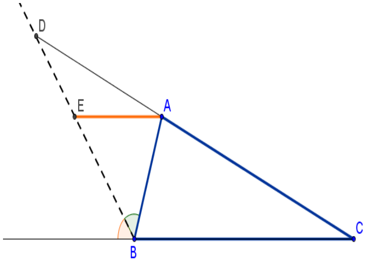
Theorem

$(BD-$ *bisectoarea exterioară a* $\angle B \Leftrightarrow \frac{DA}{DC} = \frac{AB}{BC}$

Proof.

Construim $AE || BC, E \in BD$,

Teorema bisectoarei exterioare



Theorem

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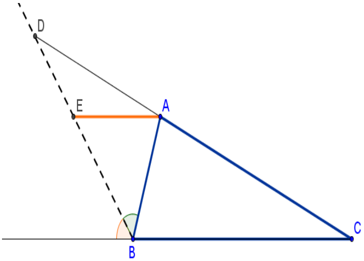
Proof.

Construim $AE || BC, E \in BD$,

$\Rightarrow \angle AEB \equiv \angle DBx (alt.int)\ si\ \angle DBA \equiv \angle DBx$

Teorema bisectoarei exterioare



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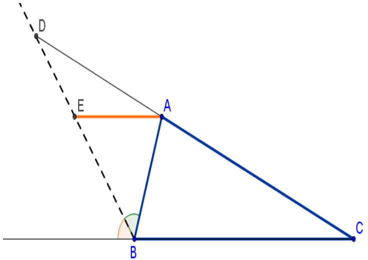
$\Rightarrow \angle AEB \equiv \angle DBx \Rightarrow \triangle AEB\ isos. \Rightarrow AE = AB$

Teorema bisectoarei exterioare



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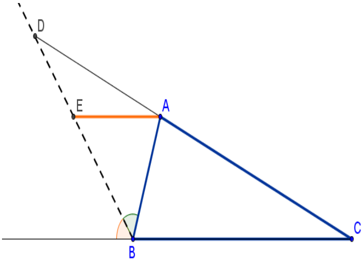
Cum $AE || BC \Rightarrow \triangle DEA \sim \triangle DBC$

Teorema bisectoarei exterioare



Theorem

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Cum $AE || BC \Rightarrow \triangle DEA \sim \triangle DBC$

$\Rightarrow \frac{DA}{DC} = \frac{EA}{BC} = \frac{AB}{BC}$

□